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Unsteady hydromagnetic free convection past a vertical flat plate

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An investigation of the unsteady boundary layer equations for an electrically conducting fluid past a semi-infinite vertical flat plate in the presence of a uniform transverse magnetic field has been carried out when the plate temperature varies suddenly in time. Expressions for the velocity and temperature distributions have been calculated in the non-dimensional forms. It is found that the skin friction decreases with increases in magnetic field.

1. Introduction

The problem of steady hydromagnetic free convection flow past a semi-infinite vertical flat plate has attracted the attention of many research workers for many years, due to its wide applications in modern technology, and a number of theoretical and experimental results have been obtained by Mori (1959), Sparrow & Cess (1961), Gupta (1962a), Lykoudis (1962), Gupta & Suryaprakasarao (1965) and D'Sa (1967). Moreover, this problem is easily amenable to experiment in a laboratory. However, the two dimensional unsteady hydromagnetic free convection boundary layer flow past a semi-infinite vertical flat plate has received considerably less attention. To the knowledge of author, the only paper to be due to Gupta (1960b), who applied the method of characteristics to study the effect of horizontal magnetic field on two dimensional unsteady laminar free convection flow past a vertical flat plate undergoing a stepwise change in temperature, has been published so far. Hence the present paper is devoted to a study of the influence of uniform magnetic field in the boundary layer flow past a vertical flat plate when the plate temperature varies suddenly in time. For solving this problem the method of similarity is applied which has its physical meaning in connection with the process of forming the boundary layer. The analysis in the present investigation is confined to low magnetic Reynolds number so that the induced magnetic field is negligible in comparison with the imposed magnetic field.

2. Fundamental Equations

Consider a semi-infinite vertical flat plate with x-axis along the plate measured from the leading edge in the direction against the gravity and y-axis normal to it. The plate is immersed in an electrically conducting fluid and a uniform magnetic field B_0 is applied along the y-axis. Then assuming that the magnetic field induced by the motion can be neglected, the dimensionless equations governing the unsteady boundary layer equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=T+\frac{\partial^2 u}{\partial y^2}-Nu, \qquad \dots (1)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial y^2}.$$

Here u and v are the velocity components in the x and y directions respectively, t the time variable, T the temperature variable, Pr the Prandtl number and $N=\sigma B_0^2 l^2/\nu\rho$ the interaction parameter. It is noted that the usual assumptions for free convection of constant properties except slight changes in density and negligible viscous dissipation are here retained. The corresponding boundary conditions are

$$\begin{array}{lll} y=0 & u=v=0 & T=T_w(x) \\ y\to\infty & u\to 0 & T\to 0 \end{array} \qquad \dots (2)$$

where $T_w(x)$ is a yet unspecified function of x.

3. Solutions of Equations

If the surface temperature is established impulsively, the initial motion is described by a balance between the viscous term and the time derivative. Define

$$u=\frac{\partial\psi}{\partial y}, \quad v=-\frac{\partial\psi}{\partial x}, \quad \psi=2\sqrt{t}f(x,\eta,t), \; \eta=y/2\sqrt{t}, \qquad \dots (3)$$

whence, provided $Pr=1$, we get

$$f_{\eta\eta\eta}+2\eta f_{\eta\eta}+4tf_{\eta t}+4t(f_x f_{\eta\eta}-f_\eta f_{\eta x})=-4tT+4Ntf_\eta, \qquad \dots (4)$$

$$T_{\eta\eta}+2\eta T_\eta-4tT_t+4t(f_x T_\eta-f_\eta T_x)=0.$$

A solution to equations (4) is sought in the form

$$T=T_0(x,\eta)+T_1(x,\eta)t^2+\dots,$$

$$f=F_0(x,\eta)t+F_1(x,\eta)t^2+F_2(x,\eta)t^3+\dots\,. \qquad \dots (5)$$

The T_k and F_k satisfy the equations

$$\begin{aligned}
&T_{0\eta\eta}+2\eta T_{0\eta}=0,\\
&F_{0\eta\eta\eta}+2\eta F_{0\eta\eta}-4F_{0\eta}=-4T_0,\\
&T_{1\eta\eta}+2\eta T_{1\eta}-8T_1=4(F_{0\eta}T_{0x}-F_{0x}T_{0\eta}),\\
&F_{1\eta\eta\eta}+2\eta F_{1\eta\eta}-8F_{1\eta}=4NF_{0\eta},\\
&F_{2\eta\eta\eta}+2\eta F_{2\eta\eta}-12F_{2\eta}=4NF_{1\eta}+4(-T_1+F_{0\eta}F_{0\eta x}-F_{0x}F_{0\eta\eta}).
\end{aligned} \qquad \dots (6)$$

For solving (6), we take

$$T_0 = T_w\theta_0(\eta), \qquad F_0 = T_w\zeta_0(\eta), \qquad T_1 = T_w\frac{dT_w}{dx}\,\theta_1(\eta),$$

$$F_1 = T_w\zeta_1(\eta), \qquad F_2 = T_w\zeta_{21}(\eta)+T_w\frac{dT_w}{dx}\,\zeta_{22}(\eta), \tag{7}$$

so that

$$\begin{gathered}\theta_0''+2\eta\theta'_0 = 0, \qquad \zeta_0'''+2\eta\zeta_0''-4\zeta_0' = -4\theta_0,\\ \theta_1''+2\eta\theta_1'-8\theta_1 = 4(\zeta'_0\theta_0-\zeta_0\theta'_0), \quad \zeta_1'''+2\eta\zeta''_1-8\zeta'_1 = 4N\zeta'_0,\\ \zeta'''_{21}+2\eta\zeta''_{21}-12\zeta'_{21} = 4N\zeta'_1,\\ \zeta'''_{22}+2\eta\zeta''_{22}-12\zeta'_{22} = 4(-\theta_1+\zeta'^2_0-\zeta_0\zeta_0''),\end{gathered} \tag{8}$$

which satisfy the following boundary conditions

$$\begin{aligned}&\eta = 0\,. && \theta_0 = 1, && \theta_1 = 0, && \zeta_i = \zeta'_i = 0,\\ &\eta \to \infty\,. && \theta_0 \to 0, && \theta_1 \to 0, && \zeta'_i \to 0.\end{aligned} \tag{9}$$

Here the dashes denote differentiation with respect to η The solutions of (8) subjected to (9) are (Pop, 1969a, b)

$$\theta_0(\eta) = \operatorname{erfc}\eta = \frac{2}{\sqrt{\pi}}\int_\eta^\infty e^{-\gamma^2}\,d\gamma,$$

$$\zeta_0'(\eta) = -2\eta^2\operatorname{erfc}\eta+\frac{2}{\sqrt{\pi}}\eta e^{-\eta^2}$$

$$\theta_1(\eta) = -\frac{2}{3}\,\eta^4\operatorname{erfc}^2\eta+\frac{1}{3\sqrt{\pi}}\Big[(\tfrac{1}{2}\eta+5\eta^3)e^{-\eta^2}+\frac{4}{5\sqrt{\pi}}\Big(1+4\eta^2+\frac{4}{3}\,\eta^4\Big)\Big]$$

$$\operatorname{erfc}\eta-\frac{1}{\sqrt{\pi}}\eta^2 e^{-2\eta^2}-\frac{4}{15\pi}\Big[1+\frac{2}{3\sqrt{\pi}}\,(5\eta+2\eta^3)\Big]e^{-\eta^2},$$

$$\zeta'_1(\eta) = -\frac{2}{3}\,N\eta^4\operatorname{efrc}\eta+\frac{1}{3\sqrt{\pi}}\,N(2\eta^3-\eta)e^{-\eta^2}, \qquad \ldots (10)$$

$$\zeta'_{21}(\eta) = -\frac{4}{45}N^2\eta^6\operatorname{erfc}\eta+\frac{2}{45\sqrt{\pi}}N^2\Big(2\eta^5-\eta^3+\frac{3}{2}\eta\Big)e^{-\eta^2},$$

$$\zeta'_{22}(\eta) = \Big(\frac{1}{8}+\frac{8}{15\pi}\Big)\Big[\frac{8}{15\pi}\Big(\frac{33}{4}\,\eta+7\eta^3+\eta^5\Big)e^{-\eta^2}-\Big(1+6\eta^2+4\eta^4+\frac{8}{15}\eta^6\Big)$$

$$\operatorname{erfc}\eta\Big]+\Big(\frac{1}{8}+\frac{3}{4}\,\eta^2+\frac{1}{4}\eta^4+\frac{1}{3}\eta^6\Big)\operatorname{erfc}^2\eta+\Big[\frac{4}{15\pi}\Big(1+4\eta^3+\frac{4}{5}\eta^5\Big)\Big]$$

$$-\frac{1}{\sqrt{\pi}}\left(\frac{1}{2}\eta+\frac{1}{3}\eta^3+\frac{2}{5}\eta^5\right)e^{-\eta^2}-\frac{8}{15\sqrt{\pi}}\eta\ \Bigg]\operatorname{erfc}\eta+$$

$$+\frac{1}{3\pi}(\eta^4-\tfrac{1}{2}\eta^2)e^{-\eta^2}+\frac{4}{15\pi}\left[1-\frac{4}{15\sqrt{\pi}}\left(\frac{8}{3}\eta+\eta^3\right)\right]e^{-\eta^2}.$$

Conclusion

Now if we consider that the surface temperature is directly proportional to a power of x (Kelleher & Yang 1968) *i.e.*, $T_w = x^n$ then the skin friction at the plate can be written, for $n = 1$, as

$$\frac{\pi^{1/2}\tau_w}{x} = t^{1/2}-0{\cdot}166\ Nt^{3/2}+(0{\cdot}033\ N^2+0{\cdot}014)t^{5/2}. \qquad \dots (11)$$

The numerical results of (11) at different times and hydromagnetic parameters N are presented in the table 1 below. It is found that the skin friction

Table 1. $\pi^{1/2}\tau_w/x$

	N			
	0	1	1 5	
0 1	0.3160	0.3109	0.3080	0.3059
0 25	0 5004	0.4807	0.4716	0.4636
0.4	0 6334	0.5938	0.5779	0.5628
0.8	0.8989	0.7994	0.7436	0.7376

decreases as the magnetic field increases. Physically this is due to the fact that the magnetic field exerts a retarding influence on the motion of the fluid which implies a reduction in the velocity gradient at the plate and consequently the skin friction is reduced.

In interpreting the results obtained in this paper, it should be borne in mind that the theory is valid for small times.

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